Lesson 1, Investigation 2, Applications Task 8 (p. 551)

a, c, d. To be completed by the student.

b. Here are the three parts of the question:

- To find the rule, look at the total number of students s. Since each class has 20 students in them, the number of classes is the number of students divided by 20 in each class. The rule is $M = \frac{s}{20}$.
- This function since it is linear it will have an inverse.
- To find the inverse, solve for s. If the school manages to exactly meet the prescribed student-to-math-class ratio, then it will be possible to calculate the number of mathematics students in the school from the number of teachers by $s = 20M$.

Lesson 1, Investigation 2, Connections Task 14 (p. 552)

a. For example, the points (0, 0) and (2, 4) are on the graph of $f(x)$ and (0, 0) and (4, 2) are on the graph of $f^{-1}(x)$.

Proof:
If (x, x^2) is on the graph of $f(x) = x^2$, then $f^{-1}(x^2) = \sqrt{x^2} = x$, so $(x^2, \sqrt{x^2}) = (x^2, x)$ is on the graph of $f^{-1}(x)$.

b. Here are the three parts of the question:

Lesson 1, Investigation 2, Extensions Task 26 (p. 556)

a, c. To be completed by the student.

b. *Hint:* Try cubic functions of the form $f(x) = x^3 \pm a$ and $g(x) = x^3 - bx$, as well as others.

Lesson 1, Investigation 2, Review Task 38 (p. 558)

a, b, d, f. To be completed by the student.

c. $7^{-3} = \frac{1}{343}$ and $7^{-2} = \frac{1}{49}$. Since $\frac{1}{343} < \frac{1}{50} < \frac{1}{49}$, the solution lies between –3 and –2. $(-3 < x < -2)$

e. $10^{2 \cdot 1} = 100$ and $10^{2 \cdot 2} = 10{,}000$. Since $100 < 3{,}000 < 10{,}000$, the solution lies between 1 and 2. $(1 < x < 2)$

Lesson 2, Investigation 1, Applications Task 2 (p. 568)

The relationship between common logarithms and exponents is, "$x = \log_{10} y$ if and only if $10^x = y$." (Recall this means, "if $x = \log_{10} y$, then $10^x = y$," and also, "if $10^x = y$, then $x = \log_{10} y$.") It will be very important to use both statements as you solve the following problems.

a. We are given $\log_{10} x = 2$, so by the first part of the definition above, $x = 10^2$, and since $10^2 = 100$, $x = 100$.

b. We are given $10^{x-5} = 60$, so by the second part of the definition, $\log_{10} 60 = x - 5$. If you add 5 to both sides of the equation, you get that $x = \log_{10} 60 + 5$.

c. To be completed by the student.

Lesson 2, Investigation 1, Applications Task 3 (p. 500)

a–d, f. To be completed by the student.

e. $15(10)^{5x+3} = 1{,}200$

$(10)^{5x+3} = \frac{1{,}200}{15}$	(1) Divide both sides of the equation by 15.
$\log 80 = 5x + 3$	(2) By definition, the logarithm finds the exponent on a base-10 number.
$\log 80 - 3 = 5x$	(3) Subtract 3 from both sides.
$\frac{\log 80 - 3}{5} = x$	(4) Divide both sides by 5 to get the value of x.

Lesson 2, Investigation 1, Applications Task 5 (p. 568)

Recall that a half-life is the time it takes for the substance being tested to be reduced to half of the original quantity. Since in this situation the radioactive iodine has 6 units, you need to find how much time it will take to reduce to 3 units.

Lesson 2, Investigation 2, Applications Task 9 (p. 569)

a. To be completed by the student.

b. Since 99% of the gas is removed, then 1% must remain. The problem does not give us an initial amount of gas, so we can pick the amount of gas we are starting with, so let's pick an easy number to work with—100 units. This way, we will start with 100 and will see how many cycles it will take to bring it down to 1 unit. Solve: $1 = 100(0.95)^n$

Lesson 2, Investigation 2, Applications Task 10 (p. 569)

a. *Hint:* Many times when working with exponential decay functions, you are looking to find how much of a substance remains. This problem is looking for how much of the light is passing through, not how much is being kept out, so the 40% that passes through is the amount we want to keep track of.

b. To be completed by the student.

Lesson 2, Investigation 2, Connections Task 15 (p. 570)

a. The problem says that it is exponential, so we know the equation is of the form $y = a \cdot b^x$. Since 5 years have gone by, the equation must be $y = a \cdot b^5$. Use the starting value of $200,000 and the selling value of $265,000 to write the equation.

b, c. To be completed by the student.

Lesson 2, Investigation 3, Applications Task 12 (p. 569)

a, d–f, h, i. To be completed by the student.

b. There are two properties being applied here:

1st: The logarithm of a product ($5x^3$) [property $\log ab = \log a + \log b$]
2nd: The logarithm of something raised to a power [property $\log ab = b \log a$]

So, $\log 5x^3 = \log 5 + 3 \log x$.

c. Use the property of the logarithm of a quotient: $\log\left(\frac{a}{b}\right) = \log a - \log b$.

$\log\left(\frac{7x}{5y}\right) = \log 7x - \log 5y = \log 7 + \log x - (\log 5 + \log y)$

g. Since the $\log a - \log \text{b} = \log\left(\frac{a}{b}\right)$, $\log x - \log 3y = \log\left(\frac{x}{3y}\right)$.

Lesson 2, Investigation 3, Reflections Task 27 (p. 573)

a. When you take the common log of a number, you are finding the exponent on base 10 that equals that number. For example, if $\log a = x$, that means $10^x = a$. We know that the exponent of a product is the sum of the exponents of the factors. For example, $10^x 10^y = 10^{x+y}$, so if you take the log of a product, that is equal to the sum of the logs.

b, d. To be completed by the student.

c. If you take the log of the number 1, regardless of the base, you will get zero, because any number raised to zero is one. Example: $10^0 = 1$.

Lesson 2, Investigation 3, Extensions Task 32 (p. 574)

a. If the distance is cut in half, the brightness will increase by a factor of 4, because S (the brightness) is inversely proportional to the square of the distance from the screen. They are related by the following formula:

$f = K_1 \log S$ (K_1 is a constant), and $S = \left(\frac{K_2}{d^2}\right)$, (where K_2 is a constant). So, $f = K_1 \log\left(\frac{K_2}{d^2}\right)$,

because we substitute $\left(\frac{K_2}{d^2}\right)$ for S in the formula. Now let the distance be half of what is was, $0.5d$.

So if the distance has been halved, then the formula is $f = K_1 \log\left(\frac{K_2}{(0.5d)^2}\right) = K_1 \log\left(\frac{K_2}{0.25d^2}\right) = K_1 \log\left(\frac{4K_2}{d^2}\right)$. Now use the property of the logarithm of a product because $f = K_1 \log\left(4\frac{K_2}{d^2}\right)$, so $f = K_1 \log 4 + K_1 \log\left(\frac{K_2}{d^2}\right)$. The question about the effect of doubling the distance to the screen is left to the student.

b, c. To be completed by the student.

Lesson 3, Investigation 1, Applications Task 1 (p. 590)

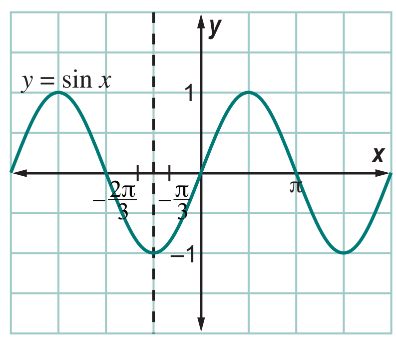
See Problem 4 on page 579 for the definition of the inverse sine function.

a. If $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$, then by the definition above the $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$.

b, c. To be completed by the student.

Lesson 3, Investigation 1, Applications Task 4 (p. 590)

a. To begin, solve the equation for sin x: $\sin x = -\frac{\sqrt{3}}{2}$. Now find out for which angles the sine function takes on the value of $-\frac{\sqrt{3}}{2}$. It happens in the 3rd and 4th quadrants where the y-coordinate is negative. Looking at the chart you made on page 582, you will see that $\sin x = -\frac{\sqrt{3}}{2}$ at $x = -\frac{\pi}{3}$. By the symmetry of the graph $y = \sin x$, $\sin x = -\frac{\sqrt{3}}{2}$ at $x = -\frac{2\pi}{3}$. See the diagram at the right. Since the sine function is periodic, it continues to take on the value $-\frac{\sqrt{3}}{2}$ every 2π radians. So, all the solutions are: $x = -\frac{\pi}{3} + 2\pi n$ or $x = -\frac{2\pi}{3} + 2\pi n$ for any integer n.



b, c. To be completed by the student.

Lesson 3, Investigation 1, Connections Task 15 (p. 593)

a. $\frac{2.4}{\sin A} = \frac{3.7}{\sin 112^\circ}$

$\sin A \approx \frac{2.4(0.9272)}{3.7} \approx 0.60$

The measure of $\angle A \approx \sin^{-1}(0.60) = 37^\circ$. This solution is unique because one of the angles given is 112°. The other angles in the triangle have to be less than 90°.

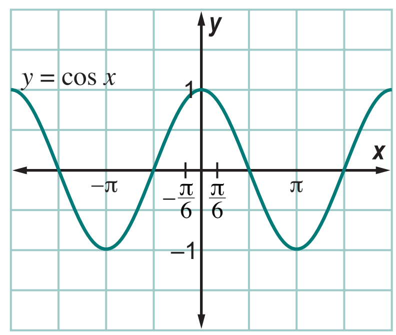
b. To be completed by the student.

Lesson 3, Investigation 2, Applications Task 6 (p. 591)

Hint: To solve these triangles without technology, you need to rely on the relationship of special right triangles. The two that you have learned in the past are 45°-45° and 30°-60° right triangles. Each of these have relationships between the lengths of their sides. The 45°-45° right triangle has two legs with the same length. If the legs have the length s, then the length of the hypotenuse is $\sqrt{2}s$. In a 30°-60° right triangle, the shorter leg has length s, the longer leg has length $\sqrt{3}s$, and the hypotenuse has length $2s$. Students may wish to draw diagrams to represent these relationships. These relationships can be found on the student's copy of Selected Key Geometric Ideas from Courses 1 and 2.

Lesson 3, Investigation 2, Applications Task 9 (p. 591)

a. To begin, solve the equation for cos x: $\cos x = \frac{\sqrt{3}}{2}$. Now find out for which angles the cosine function takes on the value of $\frac{\sqrt{3}}{2}$. It happens in the 1st and 4th quadrants where the x-coordinate is positive. The $\cos x = \frac{\sqrt{3}}{2}$ at $x = \frac{\pi}{6}$ and by symmetry the $\cos x = \frac{\sqrt{3}}{2}$ at $x = -\frac{\pi}{6}$. See the diagram at the right. Since the cosine function is periodic, it continues to take on the value $\frac{\sqrt{3}}{2}$ every 2π radians. So, all the solutions are: $x = \frac{\pi}{6} + 2\pi n$ or $x = -\frac{\pi}{6} + 2\pi n$ for any integer n.



b, c. To be completed by the student.

Lesson 3, Investigation 2, Connections Task 20 (p. 594)

a. The original equation is $a = (\sin A)\left(\frac{b}{\sin B}\right)$.

$\log\left((\sin A)\left(\frac{b}{\sin B}\right)\right) = \log(\sin A) + \log b - \log(\sin B)$. Explain why.

b. To be completed by the student.

