Examples of Tasks from ©2008 Course 1, Unit 4

Getting Started

The tasks below are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing student understanding and independence. A list of questions to use as you work together, prepared in English and Spanish, is available. Encourage students to refer to their class notes and Math Toolkit entries for assistance.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Discrete Mathematics page or the Scope and Sequence (2nd edition) might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 4

Upon completion of this unit, students should be able to:

- understand and apply Euler paths and vertex coloring.
- use vertex-edge graphs to represent and solve problems related to paths, networks, and relationships among a finite number of objects.
- model mathematically by building and using vertex-edge graph models to solve problems in a variety of settings.
- use algorithmic problem solving in designing, using, and analyzing systematic procedures for solving problems.
- use mathematical reasoning to explore and reason about properties of vertex-edge graphs.

What Solutions are Available?

Lesson 1: Investigation 1—Applications Task 1 (p. 250), Connections Task 10 (p. 255)
Investigation 2—Applications Task 4 (p. 252), Applications Task 5 (p. 252), Extensions Task 21 (p. 259)
Investigation 3—Applications Task 7 (p. 254), Reflections Task 20 (p. 258), Extensions Task 28 (p. 262)

Lesson 2: Investigation 1—Connections Task 5 (p. 278)
Investigation 2—Applications Task 1 (p. 276)

Selected Homework Tasks and Expected Solutions

(These solutions are for tasks in the 2nd edition book—2008 copyright.
For homework tasks in books with earlier copyright dates, see Helping with Homework.)

Lesson 1, Investigation 1, Applications Task 1 (p. 250)

To find the optimal path, students should start at the equipment room *G*, so they do not have to move the equipment without spraying. They should also proceed in such a way that they do not retrace their steps. There are many possible solutions. Here is one optimal path: *G* to *H*, *H* to *F*, *F* to *E*, *E* to *F*, *F* to *C*, *C* to *D*, *D* to *C*, *C* to *B*, *B* to *A*, *A* to *D*, *D* to *E*, *E* to *G*.

The second path is to be completed by the student

Lesson 1, Investigation 1, Connections Task 10 (p. 255)

a. If you count the exterior as one of the regions, this is what the table will look like for Graph I and III.

Graph	Number of Vertices (V)	Number of Regions (R)	Number of Edges (E)
I	4	4	6
II			
III	6	8	12
IV			

b–d. Using the examples above, students should fill in the rest of the table and then look for a pattern to find a rule for Part b and then apply their rule to Parts c and d to check if it is correct.

Lesson 1, Investigation 2, Applications Task 4 (p. 252)

a. To be completed by the student.

b. Graphs iii and iv have Euler circuits. (The test for an Euler circuit is that each vertex in these graphs has an *even degree*, that is, has an even number of edges terminating at each vertex. If all the vertices do not have an even degree, then there is not an Euler circuit.) To find an Euler circuit, students start anywhere. In practice, students find groups of vertices that form a circuit, within the entire graph, and then link these circuits together. A possible circuit for Graph iii is *A-B-C-D-E-F-G-H-K-L-M-N-B-C-E-F-H-K-M-N-A*. There are other circuits that will work.

c. To be completed by the student.

d. To be completed by the student.

Lesson 1, Investigation 2, Extensions Task 21 (p. 259)

a.

Graph	Sum of the Degrees of All Vertices	Number of Vertices of Odd Degree
I	30	0
II	18	2
III		
IV		

The remainder of the table is to be completed by the student.

b. The sum of the degrees and the number of odd vertices are both even numbers.

c. To be completed by the student.

Students can use the vertex-edge graph software in *CPMP-Tools* to generate more graphs, collect more data, and check their conjectures from Part b.

d. Every edge adds 2 to the sum of degrees. Thus, an easy way to obtain the sum of degrees is to count the edges and double that number. This will yield an even number.

e. For every graph, the sum of degrees is even (see Part d). The number of odd-degree vertices must be even; otherwise the sum of degrees would not be even. (Note that *odd* + *even* = *odd*, *odd* + *odd* = *even*, and *even* + *even* = *even*.)

The kind of reasoning used in this explanation is called proof by contradiction. Students do not have to know the terminology, but they should be building up their repertoire of ways to prove.

Lesson 1, Investigation 3, Applications Task 7 (p. 254)

a. The paths on the map will work as edges, so the graph could look exactly like the map, except the detail is not necessary. But the graph edges do not have to be in exactly the same places, nor the same lengths as the paths. All that is necessary for the graph model is that the edges meet at vertices just as the paths intersect at points.

b. Using the first letter of each rest area name for the vertex name, we get the following matrix.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 2 & 0 & 0 & 1 \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array}$$

The remainder of the matrix is to be completed by the student.

c. To answer this question, look at the row sums of the matrix. If any of the row sums are odd, then we know that there is not an Euler circuit because the row sum is the degree of the vertex. If the row sum is odd, so is the degree.

d. To be completed by the student.

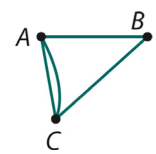
There are several ways to make every vertex even. Ask your student for two ways.

Lesson 1, Investigation 3, Reflections Task 20 (p. 258)

a. See below for graphs that correspond to matrices in parts i and ii. The matrix in part iii cannot be the adjacency matrix of a graph. A "2" in the second row, third column, and a "1" in the third row, second column, both indicate the number of edges joining vertices B and C. These numbers must be equal in an adjacency matrix.

i.

ii.



b. To be completed by the student.

Lesson 1, Investigation 3, Extensions Task 28 (p. 262)

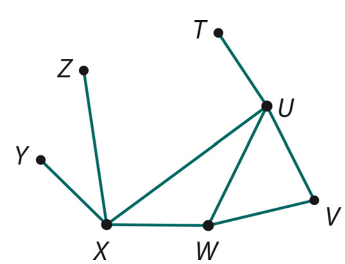
a. The degree of the vertex is 3.

b. The row sum of the first row is 2, but the degree of vertex *A* is 3. Therefore, the connection does not hold.

Lesson 2, Investigation 1, Connections Task 5 (p. 278)

a. Yes, because the relationship between vertices and edges is the same as in their previous model. (Even though the vertices are in different positions, the edges are still connecting the same vertices, which means mathematically it did not change.) Another answer could be that the radio stations connected with an edge are still 500 miles or less apart.

b. Yes. One possible redrawing is shown below.



c. Graphs I and II are planar. (Graph I can be redrawn so no edges intersect.)

Lesson 2, Investigation 2, Applications Task 1 (p. 276)

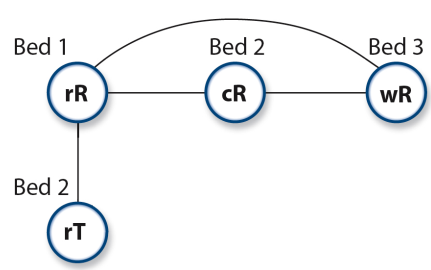
In this task, students use the method of graph coloring to solve the problem. "Coloring a vertex" does not literally mean that color has to be applied. It just means that there must be some code to keep track of the number of "colors" needed. Students could make one vertex a triangle and another a circle to indicate that they are in conflict. Or they can write "1" beside a vertex and "2" beside another. The point is that two vertices which are joined by an edge are in conflict and have to be coded differently. Since color plays a real part in this problem students may lose track of what coloring a vertex means.

a. *rR* = red roses, *cR* = coral roses, *wR* = white roses, *yT* = yellow tulips, *pT* = purple tulips, *rT* = red tulips, *yM* = yellow marigolds, *oM* = orange marigolds

b. The first thing a student must do when trying to use a graph model is to decide what the vertices will represent and what the edges will mean. In conflict problems like this one, an edge means that the two vertices connected by the edge are in conflict in some way. In this problem, some types of flowers can't be planted together (they are in conflict).

- **i.** Vertices should represent the 8 flower types (color/variety).
- **ii.** The edges should represent when flower types are of the same variety or color so that edges represent conflicting flower types.
- **iii.** The "colors" of the graph should represent the different flower beds needed.

c. Here is the *start* of a graph that will work. You can see that the roses have to be placed in three different beds, coded here as Bed 1, Bed 2, and Bed 3. The red tulips can not be placed with the red roses, so this conflict is indicated by an edge, and the red tulips could be placed in either Bed 3 or Bed 2. They have been added to Bed 2 in this graph model. As each new vertex is added students must decide where the conflicts are and then color the vertex accordingly.



d. Three beds are needed.

e. There are lots of solutions. The combinations given in this answer should go with the graph and the coloring that the student provided in Part c.

f. The conflict was having the same color or the same variety in a single bed.

